MATH201 - 800 Lab 3

Friday, September 24, 2021 8:49 AM

Second order, linear, constant coefficient eq='s.

$$ay'' + by' + cy = 0 \quad , \quad a, b, c \in \mathbb{R}.$$

Characteristic Equation: Roots of $ax^2 + bx + c = 0$ $Q(x)$

Case I: $b^2 - 4ac > 0 \Rightarrow$ two real, distinct roots

Case II: $b^2 - 4ac = 0 \Rightarrow$ one real, repeated root

Case III: $b^2 - 4ac < 0 \Rightarrow$ complex conjugates.

Case I, roots are r_1, r_2, and solution is:

$$y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}.$$

Case II, only one root r, solution is:

$$y(x) = c_1 e^{rx} + c_2 x e^{rx}$$

Case III, complex conjugates $\alpha \pm i\beta$, solution is:

$$y(x) = e^{\alpha x}[c_1 \cos(\beta x) + c_2 \sin(\beta x)]$$

Ex. solve $2y'' + 4y' - y = 0$ $\quad a = 2, b = 4, c = -1.$

Char. Eq=:

$$\frac{-4 \pm \sqrt{16 - 4(2)(-1)}}{2(2)} = \frac{-4 \pm \sqrt{24}}{4}$$

$$= -1 \pm \frac{\sqrt{6}}{2} \rightarrow \text{Case I!}$$

$$r_1 = -1 + \frac{\sqrt{6}}{2}, \quad r_2 = -1 - \frac{\sqrt{6}}{2}.$$

solution:

$$y(x) = c_1 e^{(-1 + \frac{\sqrt{6}}{2})x} + c_2 e^{(-1 - \frac{\sqrt{6}}{2})x}$$

Ex. solve $\frac{1}{2}y'' + y' + 5y = 0$, $a = 1/2, b = 1, c = 5.$

Char. Eq=:

$$\frac{-1 \pm \sqrt{1 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot 1/2} = \frac{-1 \pm \sqrt{-9}}{1}$$

$$= \underset{\substack{\alpha \\ = \\ -1}}{-1} \pm \underset{\substack{\beta \\ = \\ 3}}{3}i \rightarrow \text{Case III!}$$

$$y(x) = e^{-x}[c_1 \cos(3x) + c_2 \sin(3x)]$$

Ex. Solve $2y'' + 4y' + 2y = 0$

Char. Eq=:

$$\frac{-4 \pm \sqrt{16 - 4 \cdot 2 \cdot 2}}{2 \cdot 2} = \frac{-4 \pm 0}{4}$$

$$= -1 \quad \text{Case II!}$$

$r = -1.$

$$y(x) = c_1 e^{-x} + c_2 x e^{-x}.$$

$$y' + (1 + \tfrac{1}{t})y = 1$$

$$\mu = e^{\int(1 + \frac{1}{t})} = e^{t + \ln(t)} = te^t$$

$$\frac{d}{dt}(te^t y) = te^t$$

$$te^t y(t) = \int te^t$$